Questions

Q1.

(i) Show that $\sum_{r=1}^{16}(3 + 5r + 2^r) = 131\,798$

(4)

(ii) A sequence u_1, u_2, u_3, ... is defined by

$$u_{n+1} = \frac{1}{u_n}, \quad u_1 = \frac{2}{3}$$

Find the exact value of $\sum_{r=1}^{100} u_r$

(3)

(Total for question = 7 marks)

Q2.

A sequence of numbers a_1, a_2, a_3, ... is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \qquad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0$$

(3)

(b) For this sequence explain why $k \neq 1$

(1)

(c) Find the value of

$$\sum_{r=1}^{80} a_r$$

(3)

(Total for question = 7 marks)

Q3.

In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A company made a profit of £20 000 in its first year of trading, Year 1

A model for future trading predicts that the yearly profit will increase by 8% each year, so that the yearly profits will form a geometric sequence.

According to the model,

(a) show that the profit for Year 3 will be £23 328

(1)

(b) find the first year when the yearly profit will exceed £65 000

(3)

(c) find the total profit for the first 20 years of trading, giving your answer to the nearest £1000

(2)

(Total for question = 6 marks)

Q4.

In an arithmetic series

- the first term is 16
- the 21st term is 24

(a) Find the common difference of the series.

(2)

(b) Hence find the sum of the first 500 terms of the series.

(2)

(Total for question = 4 marks)

Q5.

Show that

$$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^\circ = \frac{9}{28}$$

(Total for question = 3 marks)

Q6.

In a geometric series the common ratio is r and sum to n terms is S_n

Given

$$S_\infty = \frac{8}{7} \times S_6$$

show that $r = \pm \frac{1}{\sqrt{k}}$, where k is an integer to be found.

(4)

(Total for question = 4 marks)

Q7.

A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre.
After the first 4 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

Using the model,

(a) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds,

(2)

(b) show that her estimated time, in minutes, to run the rth kilometre, for $5 \leq r \leq 20$, is

$$6 \times 1.05^{r-4}$$

(1)

(c) estimate the total time, in minutes and seconds, that she will take to complete the race.

(4)

(Total for question = 7 marks)

Q8.

A car has six forward gears.

The fastest speed of the car

- in 1st gear is 28 km h^{-1}
- in 6th gear is 115 km h^{-1}

Given that the fastest speed of the car in successive gears is modelled by an **arithmetic sequence**,

(a) find the fastest speed of the car in 3rd gear.

(3)

Given that the fastest speed of the car in successive gears is modelled by a **geometric sequence**,

(b) find the fastest speed of the car in 5th gear.

(3)

(Total for question = 6 marks)

Q9.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A geometric series has common ratio r and first term a.

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

(4)

Given also that S_{10} is four times S_5

(b) find the exact value of r.

(4)

(Total for question = 8 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
	(i) $\sum_{r=1}^{16}(3+5r+2^r)=131\,798$; (ii) $u_1, u_2, u_3, \ldots : u_{n+1}=\frac{1}{u_n}, u_1=\frac{2}{3}$		
(i) Way 1	$\left\{\sum_{r=1}^{16}(3+5r+2^r)=\right\} \sum_{r=1}^{16}(3+5r)+\sum_{r=1}^{16}(2^r)$	M1	3.1a
	$=\frac{16}{2}(2(8)+15(5)) + \frac{2(2^{16}-1)}{2-1}$	M1 M1	1.1b 1.1b
	$=728+131\,070 = 131\,798$ *	A1*	2.1
		(4)	
(i) Way 2	$\left\{\sum_{r=1}^{16}(3+5r+2^r)=\right\} \sum_{r=1}^{16}3 + \sum_{r=1}^{16}(5r)+\sum_{r=1}^{16}(2^r)$	M1	3.1a
	$=(3\times16)+\frac{16}{2}(2(5)+15(5)) + \frac{2(2^{16}-1)}{2-1}$	M1 M1	1.1b 1.1b
	$=48+680+131\,070 = 131\,798$ *	A1*	2.1
		(4)	
(i) Way 3	Sum = $10+17+26+39+60+97+166+299+560+1077+2106$ $+4159+8260+16457+32846+65619=131\,798$ *	M1 M1 M1 A1*	3.1a 1.1b 1.1b 2.1
		(4)	
(ii)	$\left\{u_1=\frac{2}{3}\right\}, u_2=\frac{3}{2}, u_3=\frac{2}{3}, \ldots$ *(can be implied by later working)*	M1	1.1b
	$\left\{\sum_{r=1}^{100}u_r=\right\}$ $50\left(\frac{2}{3}\right)+50\left(\frac{3}{2}\right)$ or $50\left(\frac{2}{3}+\frac{3}{2}\right)$	M1	2.2a
	$=\frac{325}{3}$ $\left(\text{or } 108\frac{1}{3} \text{ or } 108.\dot{3} \text{ or } \frac{1300}{12} \text{ or } \frac{650}{6}\right)$	A1	1.1b
		(3)	
			(7 marks)

Notes for Question	
(i)	
M1:	Uses a correct methodical strategy to enable the given sum $\sum_{r=1}^{16}(3+5r+2^r)$ to be found Allow M1 for any of the following: • expressing the given sum as either $\sum_{r=1}^{16}(3+5r)+\sum_{r=1}^{16}(2^r)$, $\sum_{r=1}^{16}3+\sum_{r=1}^{16}(5r)+\sum_{r=1}^{16}(2^r)$ or $\sum_{r=1}^{16}3+5\sum_{r=1}^{16}r+\sum_{r=1}^{16}(2^r)$ • attempting to find both $\sum_{r=1}^{16}(3+5r)$ and $\sum_{r=1}^{16}(2^r)$ separately • (3×16) and attempting to find both $\sum_{r=1}^{16}(5r)$ and $\sum_{r=1}^{16}(2^r)$ separately
M1:	**Way 1:** Correct method for finding the sum of an AP with $a=8,\ d=5,\ n=16$ **Way 2:** (3×16) and a correct method for finding the sum of an AP
M1:	Correct method for finding the sum of a GP with $a=2,\ r=2,\ n=16$
A1*:	For all steps fully shown (with correct formulae used) leading to 131 798
Note:	**Way 1:** Give 2nd M1 for writing $\sum_{r=1}^{16}(3+5r)$ as $\frac{16}{2}(8+83)$
Note:	**Way 2:** Give 2nd M1 for writing $\sum_{r=1}^{16}3+\sum_{r=1}^{16}(5r)$ as $48+\frac{16}{2}(5+80)$ or $48+680$
Note:	Give 3rd M1 for writing $\sum_{r=1}^{16}(2^r)$ as $\frac{2(1-2^{16})}{1-2}$ or $2(2^{16}-1)$ or $(2^{17}-2)$
(i)	
Way 3	
M1:	At least 6 correct terms **and** 16 terms shown
M1:	At least 10 correct terms (may not be 16 terms)
M1:	At least 15 correct terms (may not be 16 terms)
A1*:	All 16 terms correct **and** an indication that the sum is 131 798
(ii)	
M1:	For some indication that the next two terms of this sequence are $\frac{3}{2},\frac{2}{3}$
M1:	For deducing that the sum can be found by applying $50\left(\frac{2}{3}\right)+50\left(\frac{3}{2}\right)$ or $50\left(\frac{2}{3}+\frac{3}{2}\right)$, o.e.
A1:	Obtains $\frac{325}{3}$ or $108\frac{1}{3}$ or $108.\dot{3}$ or an exact equivalent
Note:	Allow 1st M1 for $u_2=\frac{3}{2}$ (or equivalent) **and** $u_3=\frac{2}{3}$ (or equivalent)
Note:	Allow 1st M1 for the first 3 terms written as $\frac{2}{3},\frac{3}{2},\frac{2}{3},\ldots$
Note:	Allow 1st M1 for the 2nd and 3rd terms written as $\frac{3}{2},\frac{2}{3},\ldots$ in the correct order
Note:	Condone $\frac{2}{3}$ written as 0.66 or awrt 0.67 for the 1st M1 mark
Note:	Give A0 for 108.3 or 108.333... without reference to $\frac{325}{3}$ or $108\frac{1}{3}$ or $108.\dot{3}$

Q2.

Question	Scheme	Marks	AOs
(a)	Uses the sequence formula $a_{n+1} = \frac{k(a_n+2)}{a_n}$ once with $a_1 = 2$	M1	1.1b
	$(a_1 = 2), a_2 = 2k, a_3 = k+1, a_4 = \frac{k(k+3)}{k+1}$ Finds four consecutive terms and sets a_4 equal to a_1 (oe)	M1	3.1a
	$\frac{k(k+3)}{k+1} = 2 \Rightarrow k^2 + 3k = 2k + 2 \Rightarrow k^2 + k - 2 = 0$ *	A1*	2.1
		(3)	
(b)	States that when $k = 1$, all terms are the same and concludes that the sequence does not have a period of order 3	B1	2.3
		(1)	
(c)	Deduces the repeating terms are $a_{1/4} = 2, a_{2/5} = -4, a_{3/6} = -1,$	B1	2.2a
	$\sum_{n=1}^{80} a_k = 26 \times (2 + -4 + -1) + 2 + -4$	M1	3.1a
	$= -80$	A1	1.1b
		(3)	
			(7 marks)
Notes:			

(a)

M1: Applies the sequence formula $a_{n+1} = \frac{k(a_n + 2)}{a_n}$ seen once.

This is usually scored in attempting to find the second term. E.g. for $a_2 = 2k$ or $a_{1+1} = \frac{k(2+2)}{2}$

M1: Attempts to find $a_1 \to a_4$ and sets $a_1 = a_4$. Condone slips.

Other methods are available. E.g. Set $a_4 = 2$, work backwards to find a_3 and equate to $k+1$

There is no requirement to see either a_1 or any of the labels. Look for the correct terms in the correct order.

There is no requirement for the terms to be simplified

FYI $a_1 = 2,\ a_2 = 2k,\ a_3 = k+1, a_4 = \frac{k(k+3)}{k+1}$ and so $2 = \frac{k(k+3)}{k+1}$

A1*: Proceeds to the given answer with accurate work showing all necessary lines. See MS for minimum

(b)

B1: States that when $k=1$, all terms are the same and concludes that the sequence does not have a period of order 3.

Do not accept "the terms just repeat" or "it would mean all the terms of the sequence are 2"

There must be some reference to the fact that it does not have order 3. Accept it has order 1.

It is acceptable to state $a_2 = a_1 = 2$ and state that the sequence does not have order 3

(c)

B1: Deduces the repeating terms are $a_{1/4} = 2, a_{2/5} = -4, a_{3/6} = -1,$

M1: Uses a clear strategy to find the sum to 80 terms. This will usually be found using multiples of the first three terms.

For example you may see $\sum_{r=1}^{80} a_r = \left(\sum_{r=1}^{78} a_r\right) + a_{79} + a_{80} = 26\times(2+-4+-1)+2+-4$

or $\sum_{r=1}^{80} a_r = \left(\sum_{r=1}^{81} a_r\right) - a_{81} = 27\times(2+-4+-1)-(-1)$

For candidates who find in terms of k award for $27\times 2 + 27\times(2k) + 26\times(k+1)$ or $80k+80$

If candidates proceed and substitute $k=-2$ into $80k+80$ to get -80 then all 3 marks are scored.

A1: -80

Note: Be aware that we have seen candidates who find the first three terms correctly, but then find

$26\frac{2}{3}\times(2+-4+-1) = 26\frac{2}{3}\times -3$ which gives the correct answer

but it is an incorrect method and should be scored B1 M0 A0

Q3.

Question	Scheme	Marks	AOs
(a)	$u_3 = £20000 \times 1.08^2 = (£)23328^*$	B1*	1.1b
		(1)	
(b)	$20000 \times 1.08^{n-1} > 65000$	M1	1.1b
	$1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \frac{\ln(3.25)}{\ln(1.08)}$ or e.g. $1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \log_{1.08}\left(\frac{13}{4}\right)$	M1	3.1b
	Year 17	A1	3.2a
		(3)	
(c)	$S_{20} = \frac{20000(1-1.08^{20})}{1-1.08}$	M1	3.4
	Awrt (£) 915 000	A1	1.1b
		(2)	
			(6 marks)
	Notes		

(a)

B1*: Uses a correct method to show that the Profit in Year 3 will be £23 328. Condone missing units

E.g. $£20000 \times 1.08^2$ or $£20000 \times 108\% \times 108\%$

This may be obtained in two steps. E.g $\frac{8}{100} \times 20000 = 1600$ followed by $\frac{8}{100} \times 21600 = 1728$ with the calculations $21600 + 1728 = 23328$ seen.

Condone calculations seen as 8% of $20000 = 1600$.

This is a show that question and the method must be seen.

It is not enough to state Year 1 = £21 600, Year 2 = £ 23 328

(b)

M1: Sets up an inequality or an equation that will allow the problem to be solved.

Allow for example N or n for $n-1$. So award for $20\,000\times1.08^{n-1}>65\,000$,

$20\,000\times1.08^{n}=65\,000$ or $20\,000\times(108\%)^{n}\geqslant65\,000$ amongst others.

Condone slips on the 20 000 and 65 000 but the 1.08 o.e. must be correct

M1: Uses a correct strategy involving logs in an attempt to solve a type of equation or inequality of the form seen above. It cannot be awarded from a sum formula

The equation/inequality must contain an index of $n-1, N, n$ etc.

Again condone slips on the 20 000 and 65 000 but additionally condone an error on the 1.08, which may appear as 1.8 for example

E.g. $20\,000\times1.08^{n}=65\,000\Rightarrow n\log1.08=\log\frac{65000}{20000}\Rightarrow n=\ldots$

E.g. $2000\times1.8^{n}=65\,000\Rightarrow\log2000+n\log1.8=\log65000\Rightarrow n=\ldots$

A1: Interprets their decimal value and gives the correct year number. Year 17

The demand of the question dictates that solutions relying entirely on calculator technology are not acceptable, BUT allow a solution that appreciates **a correct term** formula or the entire set of calculations where you may see the numbers as part of a larger list

E.g. Uses, or implies the use of, an acceptable calculation and finds value(s)

for M1: $(n=16)\Rightarrow P=20000\times1.08^{15}$ =awrt 63400 **or** $(n=17)\Rightarrow P=20000\times1.08^{16}$ = awrt 68500

M1: $(n=16)\Rightarrow P=20000\times1.08^{15}$ =awrt 63400 **and** $(n=17)\Rightarrow P=20000\times1.08^{16}$ = awrt 68500

A1: 17 years following correct method and both M's

(c)

M1: Attempts to use the model with a **correct** sum formula to find the total profit for the 20 years.

You may see an attempt to find the sum of 20 terms via a list. This is acceptable provided there are 20 terms with $u_n=1.08\times u_{n-1}$ seen at least 4 times and the sum attempted.

Condone a slip on the 20 000 (e.g appearing as 2 000) and/or a slip on the 1.08 with it being the same "r" as in (b) . Do not condone 20 appearing as 19 for instance

A1: awrt £915 000 but condone missing unit

The demand of the question dictates that all stages of working should be seen. An answer without working scores M0 A0

Q4.

Question	Scheme	Marks	AOs
(a)	$16+(21-1)\times d=24 \Rightarrow d=...$	M1	1.1b
	$d=0.4$	A1	1.1b
	Answer only scores both marks.		
		(2)	
(b)	$S_n=\frac{1}{2}n\{2a+(n-1)d\} \Rightarrow S_{500}=\frac{1}{2}\times 500\{2\times 16+499\times "0.4"\}$	M1	1.1b
	$=57\,900$	A1	1.1b
	Answer only scores both marks		
		(2)	
	(b) Alternative using $S_n=\frac{1}{2}n\{a+l\}$		
	$l=16+(500-1)\times "0.4"=215.6 \Rightarrow S_{500}=\frac{1}{2}\times 500\{16+"215.6"\}$	M1	1.1b
	$=57\,900$	A1	1.1b
			(4 marks)

Notes

(a)

M1: Correct strategy to find the common difference – must be a correct method using a = 16, and n = 21 and the 24. The method may be implied by their working.
If the AP term formula is quoted it must be correct, so use of e.g. $u_n=a+nd$ scores M0

A1: Correct value. Accept equivalents e.g. $\frac{8}{20}, \frac{4}{10}, \frac{2}{5}$ etc.

(b)

M1: Attempts to use a correct sum formula with a = 16, n = 500 and their numerical d from part (a)
If a formula is quoted it must be correct (it is in the formula book)

A1: Correct value

Alternative:

M1: Correct method for the 500th term and then uses $S_n=\frac{1}{2}n\{a+l\}$ with their l

A1: Correct value

Note that some candidates are showing implied use of $u_n=a+nd$ by showing the following:

(a) $d=\frac{24-16}{21}=\frac{8}{21}$ (b) $S_{500}=\frac{1}{2}\times 500\left\{2\times 16+499\times\frac{8}{21}\right\}=55523.80952...$

This scores (a) M0A0 (b) M1A0

Q5.

Question	Scheme	Marks	AOs
	$a=\left(\frac{3}{4}\right)^2$ or $a=\frac{9}{16}$ or $r=-\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\frac{\frac{9}{16}}{1-\left(-\frac{3}{4}\right)}=...$	M1	3.1a
	$=\frac{9}{28}$ *	A1*	1.1b
		(3)	

	Alternative 1:		
	$\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\frac{-\frac{3}{4}}{1-\left(-\frac{3}{4}\right)}=...$ or $r=-\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=-\frac{3}{7}-\left(-\frac{3}{4}\right)$	M1	3.1a
	$=\frac{9}{28}$ *	A1*	1.1b
	Alternative 2:		
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^3+\left(\frac{3}{4}\right)^4-...$	B1	2.2a
	$=\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+...-\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^5-...$ $\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+...=\left(\frac{3}{4}\right)^2\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$ or $-\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^5-...=-\left(\frac{3}{4}\right)^3\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$ $\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\left(\frac{3}{4}\right)^2\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)-\left(\frac{3}{4}\right)^3\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$	M1	3.1a
	$=\frac{9}{28}$ *	A1*	1.1b
	Alternative 3:		
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=S=\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^3+\left(\frac{3}{4}\right)^4-...$	B1	2.2a
	$=\left(\frac{3}{4}\right)^2\left(1-\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)^2-...\right)=\left(\frac{3}{4}\right)^2\left(\frac{1}{4}+S\right)\Rightarrow\frac{7}{16}S=\frac{9}{64}\Rightarrow S=...$	M1	3.1a
	$=\frac{9}{28}$ *	A1*	1.1b
			(3 marks)

Notes
B1: Deduces the correct value of the **first** term or the common ratio. The correct first term can be seen as part of them writing down the sequence but must be the **first** term. M1: Recognises that the series is infinite geometric and applies the sum to infinity GP formula with $a = \frac{9}{16}$ and $r = \pm\frac{3}{4}$ A1*: Correct proof **Alternative 1:** B1: Deduces the correct value for the sum to infinity (starting at $n = 1$) or the common ratio M1: Calculates the required value by subtracting the first term from their sum to infinity A1*: Correct proof **Alternative 2:** B1: Deduces the correct value of the **first** term or the common ratio. M1: Splits the series into "odds" and "evens", attempts the sum of both parts and calculates the required value by adding both sums A1*: Correct proof **Alternative 3:** B1: Deduces the correct value of the **first** term M1: A complete method by taking out the first term, expresses the rhs in terms of the original sum and rearranges for "S" A1*: Correct proof

Q6.

Question	Scheme	Marks	AOs
	Attempts $S_\infty = \frac{8}{7} \times S_6 \Rightarrow \frac{a}{1-r} = \frac{8}{7} \times \frac{a(1-r^6)}{1-r}$	M1	2.1
	$\Rightarrow 1 = \frac{8}{7} \times (1-r^6)$	M1	2.1
	$\Rightarrow r^6 = \frac{1}{8} \Rightarrow r = ..$	M1	1.1b
	$\Rightarrow r = \pm\frac{1}{\sqrt{2}}$ (so $k=2$)	A1	1.1b
			(4 marks)

Notes:

M1: Substitutes the correct formulae for S_∞ and S_6 into the given equation $S_\infty = \frac{8}{7} \times S_6$

M1: Proceeds to an equation just in r

M1: Solves using a correct method

A1: Proceeds to $r = \pm\frac{1}{\sqrt{2}}$ giving $k = 2$

Q7.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$24 + (6 \times 1.05) + (6 \times 1.05^2)$ minutes	M1	This mark is for a method to find the time taken for the competitor to run 6 km
	= 96.915 minutes = 36 minutes 55 seconds	A1	This mark is given for finding the total time as required
(b)	For example, 5th km = 6×1.05^1 6th km = 6×1.05^2 7th km = 6×1.05^3 … rth km = $6 \times 1.05^{r-4}$	B1	This mark is given for showing the time taken to run the rth km, as required
(c)	$24 + \sum_{r=5}^{20} 6 \times 1.05^{r-4}$	M1	This mark is given for showing the total time to run the race is the time taken for the first 4 km added to the time taken from 5th to 20th km
	$= 24 + 6.3 \times \frac{(1.05^{16} - 1)}{1.05 - 1}$	M1	This mark is given for using $s = a\left(\frac{1-r^n}{1-r}\right)$ where $a = 6 \times 1.05 = 6.3$, $r = 1.05$ and $n = 20 - 4 = 16$
	= 24 + 149.04	A1	This mark is given for a correct total time (represented decimally)
	= 173 minutes and 3 seconds	A1	This mark is given for finding a correct total time given in minutes in seconds

Q8.

Question	Scheme	Marks	AOs
(a)	Uses $115 = 28 + 5d \Rightarrow d = (17.4)$	M1	3.1b
	Uses $28 + 2\times\text{"}17.4\text{"} = ...$	M1	3.4
	$= 62.8$ (km h $^{-1}$)	A1	1.1b
		(3)	
(b)	Uses $115 = 28r^5 \Rightarrow r = (1.3265)$	M1	3.1b
	Uses $28\times\text{"}1.3265^4\text{"} = ...$ or $\frac{115}{\text{"}1.3265\text{"}}$	M1	3.4
	$= 86.7$ (km h $^{-1}$)	A1	1.1b
		(3)	
			(6 marks)
Notes:			

(a)

M1: Translates the problem into maths using n^{th} term $= a + (n-1)d$ and attempts to find d

Look for either $115 = 28 + 5d \Rightarrow d = ...$ or an attempt at $\frac{115-28}{5}$ condoning slips

It is implied by use of $d = 17.4$ Note that $115 = 28 + 6d \Rightarrow d = ...$ is M0

M1: Uses the model to find the fastest speed the car can go in 3rd gear using $28 + 2\text{"}d\text{"}$ or equivalent.
This can be awarded following an incorrect method of finding "d"

A1: 62.8 km/ h Lack of units are condoned. Allow exact alternatives such as $\frac{314}{5}$

(b)

M1: Translates the problem into maths using n^{th} term $= ar^{n-1}$ and attempts to find r
It must use the 1st and 6th gear and not the 3rd gear found in part (a)

Look for either $115 = 28r^5 \Rightarrow r = ...$ o.e. or $\sqrt[5]{\frac{115}{28}}$ condoning slips.

It is implied by stating or using $r =$ awrt 1.33

M1: Uses the model to find the fastest speed the car can go in 5th gear using $28\times\text{"}r^4\text{"}$ or $\frac{115}{\text{"}r\text{"}}$ o.e.

This can be awarded following an incorrect method of finding "r"
A common misread seems to be finding the fastest speed the car can go in 3rd gear as in (a).
Providing it is clear what has been done, e.g. $u_3 = 28\times\text{"}r^2\text{"}$ it can be awarded this mark.

A1: awrt 86.7 km/h Lack of units are condoned. Expressions must be evaluated.

Q9.

Question	Scheme	Marks	AOs
(a)	$S_n = a + ar + ar^2 + ... + ar^{n-1}$	B1	1.2
	$rS_n = ar + ar^2 + ar^3 + ... + ar^n \Rightarrow S_n - rS_n = ...$	M1	2.1
	$S_n - rS_n = a - ar^n$	A1	1.1b
	$S_n(1-r) = a(1-r^n) \Rightarrow S_n = \frac{a(1-r^n)}{(1-r)}$ *	A1*	2.1
		(4)	
(b)	$\frac{a(1-r^{10})}{1-r} = 4\times\frac{a(1-r^5)}{1-r}$ or $4\times\frac{a(1-r^{10})}{1-r} = \frac{a(1-r^5)}{1-r}$ Equation in r^{10} and r^5 (and possibly $1-r$)	M1	3.1a
	$1 - r^{10} = 4(1-r^5)$	A1	1.1b
	$r^{10} - 4r^5 + 3 = 0 \Rightarrow (r^5-1)(r^5-3) = 0 \Rightarrow r^5 = ...$ or e.g. $1-r^{10} = 4(1-r^5) \Rightarrow (1-r^5)(1+r^5) = 4(1-r^5) \Rightarrow r^5 = ...$	dM1	2.1
	$r = \sqrt[5]{3}$ oe only	A1	1.1b
		(4)	
			(8 marks)

Notes:

(a)

B1: Writes out the sum or lists terms. Condone the omission of *S*.
The sum must include the first and last terms and (at least) two other correct terms and no incorrect terms e.g. ar^n
Note that the sum may be seen embedded within their working.

M1: For the key step in attempting to multiply the first series by *r* and subtracting.

A1: $S_n - rS_n = a - ar^n$ either way around but condone one side being prematurely factorised (but not both) **following correct work but this could follow B0 if insufficient terms were shown.**

A1*: Depends on all previous marks. Proceeds to given result showing all steps including seeing both sides unfactorised at some point in their working.
Note: If terms are listed rather than added then allow the first 3 marks if otherwise correct but withhold the final mark.

(b)

M1: For the correct strategy of producing an equation in just r^{10} and r^5 (and possibly $(1-r)$) with the "4" on either side using the result from part (a) and makes progress to at least cancel through by *a*
Some candidates retain the "$1-r$" and start multiplying out e.g. $(1-r)(1-r^{10})$ and this mark is still available as long as there is progress in cancelling the "*a*".

A1: Correct equation with the *a* and the $1-r$ cancelled. Allow any correct equation in just r^5 and r^{10}

dM1: Depends on the first M. Solves as far as $r^5 = ...$ by solving a 3 term quadratic in r^5 by a valid method – see general guidance or by difference of 2 squares – see above

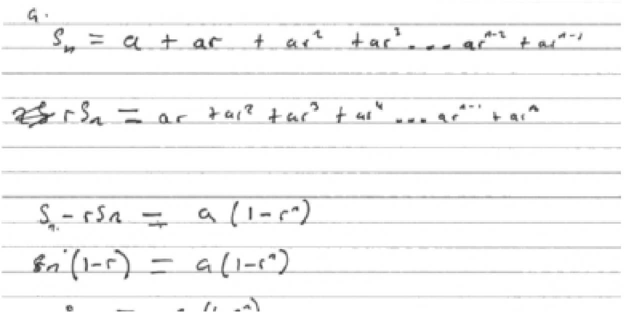
A1: $r = \sqrt[5]{3}$ oe only. The solution $r = 1$ if found must be rejected here.

(b) Note: For candidates who use $S_5 = 4S_{10}$ expect to see:

$$4\times\frac{a\left(1-r^{10}\right)}{1-r}=\frac{a\left(1-r^{5}\right)}{1-r}\Rightarrow 4\left(1-r^{10}\right)=\left(1-r^{5}\right) \text{ M1A0}$$

$$4r^{10}-r^{5}-3=0\Rightarrow\left(4r^{5}+3\right)\left(r^{5}-1\right)=0\Rightarrow r^{5}=\ldots \text{ or } 4\left(1-r^{5}\right)\left(1+r^{5}\right)=\left(1-r^{5}\right)\Rightarrow r^{5}=\ldots \text{dM1A0}$$

Example for (a)



This scores B1M1A1A0:

B1: Writes down the sum including first and last terms and at least 2 other correct terms and no incorrect terms

M1: Multiplies by *r* and subtracts

A1: Correct equation (we allow one side to be prematurely factorised)

A0: One side was prematurely factorised